5.3.1. Metoda supstitucije za neodređene integrale

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Fotografija: Jeremy Thomas

Metoda supstitucije za neodređene integrale

Neka su zadane funkcije $f : D_1 \subseteq \mathbb{R} \to \mathbb{R}$ i $g : D_2 \subseteq \mathbb{R} \to \mathbb{R}$. Pretpostavimo da je

$$\int f(x)\,\mathrm{d}x = F(x) + C$$

(tj. $F' = f$). Tada smijemo računati ovako:

$$\begin{aligned}
\int f(g(x)) \cdot g'(x)\,\mathrm{d}x &= \begin{bmatrix} t = g(x) \\ \mathrm{d}t = g'(x)\,\mathrm{d}x \end{bmatrix} \\
&= \int f(t)\,\mathrm{d}t \\
&= F(t) + C \\
&= F(g(x)) + C.
\end{aligned}$$

(U prvoj jednakosti **uvodimo supstituciju**, a u zadnjoj se **vraćamo iz supstitucije**.)

Metoda supstitucije za neodređene integrale

Neka su zadane funkcije $f : D_1 \subseteq \mathbb{R} \to \mathbb{R}$ i $g : D_2 \subseteq \mathbb{R} \to \mathbb{R}$. Pretpostavimo da je

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$$\begin{aligned}\int f(g(x)) \cdot g'(x)\,\mathrm{d}x &= \begin{bmatrix} t = g(x) \\ \mathrm{d}t = g'(x)\,\mathrm{d}x \end{bmatrix} \\ &= \int f(t)\,\mathrm{d}t \\ &= F(t) + C \\ &= F(g(x)) + C.\end{aligned}$$

(U prvoj jednakosti **uvodimo supstituciju**, a u zadnjoj se **vraćamo iz supstitucije**.)

Dokaz. $F(g(x))' = F'(g(x)) \cdot g'(x) = f(g(x)) \cdot g'(x)$.

Primjer 1(a)

Izračunajte integral $\int (3x+2)^3 \, \mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \, \mathrm{d}x = e^x + C$$
$$\int a^x \, \mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0, \ a \neq 1)$$
$$\int \cos x \, \mathrm{d}x = \sin x + C$$
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$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Primjer 1(a)

Izračunajte integral $\int (3x+2)^3\,\mathrm{d}x$.

Rješenje. Imamo

$$\int (3x+2)^3\,\mathrm{d}x = \begin{bmatrix} t = 3x+2 \\ \mathrm{d}t = 3\,\mathrm{d}x \;\Rightarrow\; \mathrm{d}x = \frac{\mathrm{d}t}{3} \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$
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$$= \int t^3 \cdot \frac{\mathrm{d}t}{3}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
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$$= \int t^3 \cdot \frac{\mathrm{d}t}{3}$$

$$= \frac{1}{3} \cdot \frac{t^4}{4} + C$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

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$$= \int t^3 \cdot \frac{\mathrm{d}t}{3}$$

$$= \frac{1}{3} \cdot \frac{t^4}{4} + C$$

$$= \frac{t^4}{12} + C$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

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$$= \int t^3 \cdot \frac{\mathrm{d}t}{3}$$
$$= \frac{1}{3}\cdot\frac{t^4}{4} + C$$
$$= \frac{t^4}{12} + C$$
$$= \frac{(3x+2)^4}{12} + C.$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
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Primjer 1(b)

Izračunajte integral $\int x^2 \left(2x^3+4\right)^2 \,\mathrm{d}x.$

$$\int \mathrm{d}x = x + C$$

$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

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$$\int e^x \,\mathrm{d}x = e^x + C$$

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Primjer 1(b)

Izračunajte integral $\int x^2\left(2x^3+4\right)^2\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x^2\left(2x^3+4\right)^2\,\mathrm{d}x=\left[\begin{array}{c} t=2x^3+4 \\ \mathrm{d}t=6x^2\,\mathrm{d}x \quad\Rightarrow\quad x^2\,\mathrm{d}x=\frac{\mathrm{d}t}{6}\end{array}\right]$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x=\frac{x^{a+1}}{a+1}+C \quad (a\neq -1)$$

$$\int \frac{\mathrm{d}x}{x}=\ln|x|+C$$

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Rješenje. Imamo

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Primjer 1(b)

Izračunajte integral $\int x^2\left(2x^3+4\right)^2\,\mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}
\int x^2\left(2x^3+4\right)^2\,\mathrm{d}x &= \left[\begin{array}{c} t=2x^3+4 \\ \mathrm{d}t=6x^2\,\mathrm{d}x \;\Rightarrow\; x^2\,\mathrm{d}x=\frac{\mathrm{d}t}{6}\end{array}\right] \\
&= \int t^2\cdot\frac{\mathrm{d}t}{6} \\
&= \frac{1}{6}\cdot\frac{t^3}{3}+C
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1}+C \quad (a\neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
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Primjer 1(b)

Izračunajte integral $\displaystyle\int x^2\left(2x^3+4\right)^2\,\mathrm{d}x.$

Rješenje. Imamo

$$\begin{aligned}
\int x^2\left(2x^3+4\right)^2\,\mathrm{d}x &= \begin{bmatrix} t=2x^3+4 \\ \mathrm{d}t=6x^2\,\mathrm{d}x \quad \Rightarrow \quad x^2\,\mathrm{d}x=\frac{\mathrm{d}t}{6}\end{bmatrix} \\
&= \int t^2\cdot\frac{\mathrm{d}t}{6} \\
&= \frac{1}{6}\cdot\frac{t^3}{3}+C \\
&= \frac{\left(2x^3+4\right)^3}{18}+C.
\end{aligned}$$

$$\int \mathrm{d}x = x+C$$

$$\int x^a\,\mathrm{d}x=\frac{x^{a+1}}{a+1}+C \quad (a\neq -1)$$

$$\int \frac{\mathrm{d}x}{x}=\ln|x|+C$$

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Primjer 1(c)

Izračunajte integral $\int x\, e^{5x^2+4}\,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
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Primjer 1(c)

Izračunajte integral $\int x\, e^{5x^2+4}\, \mathrm{d}x$.

Rješenje. Imamo

$$\int x\, e^{5x^2+4}\, \mathrm{d}x = \begin{bmatrix} t = 5x^2 + 4 \\ \mathrm{d}t = 10x\, \mathrm{d}x \quad \Rightarrow \quad x\, \mathrm{d}x = \frac{\mathrm{d}t}{10} \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\, \mathrm{d}x = e^x + C$$

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Primjer 1(c)

Izračunajte integral $\int x\, e^{5x^2+4}\, dx$.

Rješenje. Imamo

$$\int x\, e^{5x^2+4}\, dx = \left[\begin{array}{c} t = 5x^2+4 \\ dt = 10x\, dx \quad \Rightarrow \quad x\, dx = \frac{dt}{10} \end{array}\right]$$
$$= \int e^t \cdot \frac{dt}{10}$$

$$\int dx = x + C$$
$$\int x^a\, dx = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{dx}{x} = \ln|x| + C$$
$$\int e^x\, dx = e^x + C$$
$$\int a^x\, dx = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\, dx = \sin x + C$$
$$\int \sin x\, dx = -\cos x + C$$
$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$
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$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Primjer 1(c)

Izračunajte integral $\int x\, e^{5x^2+4}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x\, e^{5x^2+4}\,\mathrm{d}x = \begin{bmatrix} t = 5x^2+4 \\ \mathrm{d}t = 10x\,\mathrm{d}x \;\Rightarrow\; x\,\mathrm{d}x = \frac{\mathrm{d}t}{10} \end{bmatrix}$$

$$= \int e^t \cdot \frac{\mathrm{d}t}{10}$$

$$= \frac{1}{10} e^t + C$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\; a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

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Primjer 1(c)

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Rješenje. Imamo

$$\begin{aligned}
\int x\, e^{5x^2+4}\, \mathrm{d}x &= \begin{bmatrix} t = 5x^2 + 4 \\ \mathrm{d}t = 10x\, \mathrm{d}x \quad \Rightarrow \quad x\, \mathrm{d}x = \frac{\mathrm{d}t}{10} \end{bmatrix} \\
&= \int e^t \cdot \frac{\mathrm{d}t}{10} \\
&= \frac{1}{10} e^t + C \\
&= \frac{1}{10} e^{5x^2+4} + C.
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\, \mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

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$$\int \sin x\, \mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(a)

Izračunajte integral $\int \sqrt[4]{7x-16}\,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(a)

Izračunajte integral $\int \sqrt[4]{7x-16}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int \sqrt[4]{7x-16}\,\mathrm{d}x = \begin{bmatrix} t = 7x-16 \\ \mathrm{d}t = 7\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{\mathrm{d}t}{7} \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(a)

Izračunajte integral $\int \sqrt[4]{7x-16}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int \sqrt[4]{7x-16}\,\mathrm{d}x = \left[\begin{array}{c} t = 7x-16 \\ \mathrm{d}t = 7\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{\mathrm{d}t}{7} \end{array}\right]$$

$$= \int \sqrt[4]{t} \cdot \frac{\mathrm{d}t}{7}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(a)

Izračunajte integral $\int \sqrt[4]{7x-16}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int \sqrt[4]{7x-16}\,\mathrm{d}x = \begin{bmatrix} t = 7x-16 \\ \mathrm{d}t = 7\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{\mathrm{d}t}{7} \end{bmatrix} = \int \sqrt[4]{t}\cdot\frac{\mathrm{d}t}{7} = \frac{1}{7}\int t^{\frac{1}{4}}\,\mathrm{d}t$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(a)

Izračunajte integral $\int \sqrt[4]{7x-16}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int \sqrt[4]{7x-16}\,\mathrm{d}x = \left[\begin{array}{c} t = 7x-16 \\ \mathrm{d}t = 7\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{\mathrm{d}t}{7} \end{array}\right]$$
$$= \int \sqrt[4]{t} \cdot \frac{\mathrm{d}t}{7}$$
$$= \frac{1}{7}\int t^{\frac{1}{4}}\,\mathrm{d}t$$
$$= \frac{1}{7} \cdot \frac{t^{\frac{5}{4}}}{\frac{5}{4}} + C$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(a)

Izračunajte integral $\displaystyle\int \sqrt[4]{7x-16}\,\mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}
\int \sqrt[4]{7x-16}\,\mathrm{d}x &= \left[\begin{array}{c} t = 7x-16 \\ \mathrm{d}t = 7\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{\mathrm{d}t}{7} \end{array}\right] \\
&= \int \sqrt[4]{t}\cdot\frac{\mathrm{d}t}{7} \\
&= \frac{1}{7}\int t^{\frac{1}{4}}\,\mathrm{d}t \\
&= \frac{1}{7}\cdot\frac{t^{\frac{5}{4}}}{\frac{5}{4}} + C \\
&= \frac{4}{35}(7x-16)^{\frac{5}{4}} + C.
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(b)

Izračunajte integral $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{5x-2}}$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x \,\mathrm{d}x = e^x + C$$
$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x \,\mathrm{d}x = \sin x + C$$
$$\int \sin x \,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(b)

Izračunajte integral $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{5x-2}}$.

Rješenje. Imamo

$$\int \frac{\mathrm{d}x}{\sqrt{5x-2}} = \begin{bmatrix} t = 5x-2 \\ \mathrm{d}t = 5\,\mathrm{d}x \;\Rightarrow\; \mathrm{d}x = \frac{\mathrm{d}t}{5} \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\; a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(b)

Izračunajte integral $\int \frac{\mathrm{d}x}{\sqrt{5x-2}}$.

Rješenje. Imamo

$$\int \frac{\mathrm{d}x}{\sqrt{5x-2}} = \begin{bmatrix} t = 5x-2 \\ \mathrm{d}t = 5\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{\mathrm{d}t}{5} \end{bmatrix}$$

$$= \int \frac{1}{\sqrt{t}} \cdot \frac{\mathrm{d}t}{5}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(b)

Izračunajte integral $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{5x-2}}$.

Rješenje. Imamo

$$\begin{aligned}
\int \frac{\mathrm{d}x}{\sqrt{5x-2}} &= \begin{bmatrix} t = 5x - 2 \\ \mathrm{d}t = 5\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{\mathrm{d}t}{5} \end{bmatrix} \\
&= \int \frac{1}{\sqrt{t}} \cdot \frac{\mathrm{d}t}{5} \\
&= \frac{1}{5} \int t^{-\frac{1}{2}}\,\mathrm{d}t
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(b)

Izračunajte integral $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{5x-2}}$.

Rješenje. Imamo

$$\begin{aligned}
\int \frac{\mathrm{d}x}{\sqrt{5x-2}} &= \left[\begin{array}{c} t = 5x-2 \\ \mathrm{d}t = 5\,\mathrm{d}x \ \Rightarrow \ \mathrm{d}x = \frac{\mathrm{d}t}{5} \end{array}\right] \\
&= \int \frac{1}{\sqrt{t}} \cdot \frac{\mathrm{d}t}{5} \\
&= \frac{1}{5} \int t^{-\frac{1}{2}}\,\mathrm{d}t \\
&= \frac{1}{5} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(b)

Izračunajte integral $\int \frac{\mathrm{d}x}{\sqrt{5x-2}}$.

Rješenje. Imamo

$$
\begin{aligned}
\int \frac{\mathrm{d}x}{\sqrt{5x-2}} &= \left[\begin{array}{c} t = 5x-2 \\ \mathrm{d}t = 5\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{\mathrm{d}t}{5} \end{array}\right] \\
&= \int \frac{1}{\sqrt{t}} \cdot \frac{\mathrm{d}t}{5} \\
&= \frac{1}{5} \int t^{-\frac{1}{2}}\,\mathrm{d}t \\
&= \frac{1}{5} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C \\
&= \frac{2}{5}\sqrt{5x-2} + C.
\end{aligned}
$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(c)

Izračunajte integral $\int x^2\sqrt{6x^3+4}\,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(c)

Izračunajte integral $\int x^2\sqrt{6x^3+4}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x^2\sqrt{6x^3+4}\,\mathrm{d}x = \begin{bmatrix} t = 6x^3+4 \\ \mathrm{d}t = 18x^2\,\mathrm{d}x \quad\Rightarrow\quad x^2\,\mathrm{d}x = \frac{\mathrm{d}t}{18} \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(c)

Izračunajte integral $\int x^2\sqrt{6x^3+4}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x^2\sqrt{6x^3+4}\,\mathrm{d}x = \left[\begin{array}{c} t = 6x^3+4 \\ \mathrm{d}t = 18x^2\,\mathrm{d}x \quad\Rightarrow\quad x^2\,\mathrm{d}x = \frac{\mathrm{d}t}{18} \end{array}\right]$$

$$= \int \sqrt{t}\cdot\frac{\mathrm{d}t}{18}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(c)

Izračunajte integral $\int x^2\sqrt{6x^3+4}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int x^2\sqrt{6x^3+4}\,\mathrm{d}x = \left[\begin{array}{c} t = 6x^3+4 \\ \mathrm{d}t = 18x^2\,\mathrm{d}x \quad\Rightarrow\quad x^2\,\mathrm{d}x = \frac{\mathrm{d}t}{18} \end{array}\right]$$

$$= \int \sqrt{t}\cdot\frac{\mathrm{d}t}{18}$$

$$= \frac{1}{18}\int t^{\frac{1}{2}}\,\mathrm{d}t$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(c)

Izračunajte integral $\int x^2\sqrt{6x^3+4}\,\mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}
\int x^2\sqrt{6x^3+4}\,\mathrm{d}x &= \left[\begin{array}{c} t = 6x^3+4 \\ \mathrm{d}t = 18x^2\,\mathrm{d}x \quad \Rightarrow \quad x^2\,\mathrm{d}x = \frac{\mathrm{d}t}{18} \end{array}\right] \\
&= \int \sqrt{t}\cdot\frac{\mathrm{d}t}{18} \\
&= \frac{1}{18}\int t^{\frac{1}{2}}\,\mathrm{d}t \\
&= \frac{1}{18}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(c)

Izračunajte integral $\int x^2\sqrt{6x^3+4}\,\mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}\int x^2\sqrt{6x^3+4}\,\mathrm{d}x &= \left[\begin{array}{c} t=6x^3+4 \\ \mathrm{d}t = 18x^2\,\mathrm{d}x \quad\Rightarrow\quad x^2\,\mathrm{d}x = \frac{\mathrm{d}t}{18}\end{array}\right] \\ &= \int \sqrt{t}\cdot\frac{\mathrm{d}t}{18} \\ &= \frac{1}{18}\int t^{\frac{1}{2}}\,\mathrm{d}t \\ &= \frac{1}{18}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C \\ &= \frac{1}{27}\left(6x^3+4\right)^{\frac{3}{2}}+C.\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a\neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a>0,\ a\neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a>0)$$

Zadatak 47(d)

Izračunajte integral $\displaystyle\int \frac{\mathrm{d}x}{\cos^2 x \cdot \operatorname{tg} x}$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(d)

Izračunajte integral $\displaystyle\int \frac{\mathrm{d}x}{\cos^2 x \cdot \operatorname{tg} x}$.

Rješenje. Imamo

$$\int \frac{\mathrm{d}x}{\cos^2 x \cdot \operatorname{tg} x} = \begin{bmatrix} t = \operatorname{tg} x \\ \mathrm{d}t = \frac{1}{\cos^2 x}\,\mathrm{d}x \end{bmatrix}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(d)

Izračunajte integral $\displaystyle\int \frac{\mathrm{d}x}{\cos^2 x \cdot \operatorname{tg} x}$.

Rješenje. Imamo

$$\int \frac{\mathrm{d}x}{\cos^2 x \cdot \operatorname{tg} x} = \left[\begin{array}{c} t = \operatorname{tg} x \\ \mathrm{d}t = \frac{1}{\cos^2 x}\,\mathrm{d}x \end{array}\right] = \int \frac{\mathrm{d}t}{t}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(d)

Izračunajte integral $\displaystyle\int \frac{\mathrm{d}x}{\cos^2 x \cdot \operatorname{tg} x}$.

Rješenje. Imamo

$$
\begin{aligned}
\int \frac{\mathrm{d}x}{\cos^2 x \cdot \operatorname{tg} x} &= \begin{bmatrix} t = \operatorname{tg} x \\ \mathrm{d}t = \frac{1}{\cos^2 x}\,\mathrm{d}x \end{bmatrix} \\
&= \int \frac{\mathrm{d}t}{t} \\
&= \ln|t| + C
\end{aligned}
$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(d)

Izračunajte integral $\displaystyle\int \frac{\mathrm{d}x}{\cos^2 x \cdot \operatorname{tg} x}$.

Rješenje. Imamo

$$\begin{aligned}\int \frac{\mathrm{d}x}{\cos^2 x \cdot \operatorname{tg} x} &= \begin{bmatrix} t = \operatorname{tg} x \\ \mathrm{d}t = \frac{1}{\cos^2 x}\,\mathrm{d}x \end{bmatrix} \\ &= \int \frac{\mathrm{d}t}{t} \\ &= \ln|t| + C \\ &= \ln|\operatorname{tg} x| + C.\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(e)

Izračunajte integral $\displaystyle\int \frac{x^2}{1+x^6}\,\mathrm{d}x$.

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(e)

Izračunajte integral $\displaystyle\int \frac{x^2}{1+x^6}\,\mathrm{d}x$.

Rješenje. Imamo

$$\int \frac{x^2}{1+x^6}\,\mathrm{d}x = \left[\begin{array}{c} t = x^3 \\ \mathrm{d}t = 3x^2\,\mathrm{d}x \quad\Rightarrow\quad x^2\,\mathrm{d}x = \frac{\mathrm{d}t}{3} \end{array}\right]$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(e)

Izračunajte integral $\displaystyle\int \frac{x^2}{1+x^6}\,\mathrm{d}x.$

Rješenje. Imamo

$$\int \frac{x^2}{1+x^6}\,\mathrm{d}x = \begin{bmatrix} t = x^3 \\ \mathrm{d}t = 3x^2\,\mathrm{d}x \quad \Rightarrow \quad x^2\,\mathrm{d}x = \frac{\mathrm{d}t}{3} \end{bmatrix}$$

$$= \int \frac{1}{1+t^2}\,\frac{\mathrm{d}t}{3}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(e)

Izračunajte integral $\displaystyle\int \frac{x^2}{1+x^6}\,\mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}
\int \frac{x^2}{1+x^6}\,\mathrm{d}x &= \left[\begin{array}{c} t = x^3 \\ \mathrm{d}t = 3x^2\,\mathrm{d}x \quad\Rightarrow\quad x^2\,\mathrm{d}x = \frac{\mathrm{d}t}{3} \end{array}\right] \\
&= \int \frac{1}{1+t^2}\,\frac{\mathrm{d}t}{3} \\
&= \frac{1}{3}\operatorname{arctg} t + C
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$

$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x\,\mathrm{d}x = e^x + C$$

$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\ a \neq 1)$$

$$\int \cos x\,\mathrm{d}x = \sin x + C$$

$$\int \sin x\,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C \quad (a > 0)$$

Zadatak 47(e)

Izračunajte integral $\displaystyle\int \frac{x^2}{1+x^6}\,\mathrm{d}x$.

Rješenje. Imamo

$$\begin{aligned}
\int \frac{x^2}{1+x^6}\,\mathrm{d}x &= \begin{bmatrix} t = x^3 \\ \mathrm{d}t = 3x^2\,\mathrm{d}x \;\Rightarrow\; x^2\,\mathrm{d}x = \frac{\mathrm{d}t}{3} \end{bmatrix} \\
&= \int \frac{1}{1+t^2}\,\frac{\mathrm{d}t}{3} \\
&= \frac{1}{3}\,\mathrm{arctg}\, t + C \\
&= \frac{1}{3}\,\mathrm{arctg}\left(x^3\right) + C.
\end{aligned}$$

$$\int \mathrm{d}x = x + C$$
$$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C \quad (a \neq -1)$$
$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$
$$\int e^x\,\mathrm{d}x = e^x + C$$
$$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C \quad (a > 0,\; a \neq 1)$$
$$\int \cos x\,\mathrm{d}x = \sin x + C$$
$$\int \sin x\,\mathrm{d}x = -\cos x + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \mathrm{tg}\, x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\mathrm{ctg}\, x + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$
$$\int \frac{\mathrm{d}x}{1+x^2} = \mathrm{arctg}\, x + C$$
$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a} + C \quad (a > 0)$$